

1)

a) $N(x) = f(x)g(x)$

$$N'(x) = f'(x)g(x) + g'(x)f(x)$$

$$N'(-1) = f'(-1)g(-1) + g'(-1)f(-1)$$

$$f(-1) = 6$$

$$f'(-1) = -2$$

$$g(-1) = (-4)^{1/3}$$

$$g'(x) = \frac{1}{3}(x^2 + 5x)^{-2/3}(2x + 5)$$

$$g'(-1) = \frac{1}{3}(-4)^{-2/3}(3)$$

b) $P(x) = \frac{g(x)}{5f(x)}$

$$P'(x) = \frac{5f(x)g'(x) - 5g(x)f'(x)}{[5f(x)]^2}$$

$$P'(4) = \frac{5f(4)g'(4) - 5g(4)f'(4)}{[5f(4)]^2}$$

$$f(4) = 3$$

$$f'(4) = \frac{1}{3}$$

$$g(4) = (36)^{1/3}$$

$$g'(4) = \frac{1}{3}(36)^{-2/3}(13)$$

* NOTE: You could have used a calculator to solve parts (a) & (b)

c) $h(x) = 7\cos x + x^3$

$$h'(x) = -7\sin x + 3x^2$$

$$f'(x) = -2$$

$$-7\sin x + 3x^2 = -2$$

$$x = 0.342$$

$$2) a) RDC_{avg} = \frac{f(5) - f(-5)}{5 - (-5)}$$

$$f'(x) = \frac{3}{5} \text{ twice}$$

$$= \frac{9 - 3}{10} = \frac{3}{5}$$

$$b) f(2) = 6 \quad f'(2) = -2$$

$$y - 6 = -2(x - 2)$$

$$c) \lim_{x \rightarrow -1} \frac{f(x) - f(-1)}{x - (-1)} = f'(-1) = 4$$

$$\lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3} = f'(3) \text{ is undefined}$$

$$\lim_{x \rightarrow 3^-} \frac{f(x) - f(3)}{x - 3} \neq \lim_{x \rightarrow 3^+} \frac{f(x) - f(3)}{x - 3}$$

$$d) g(x) = f(x) \cdot \ln x$$

$$g'(x) = f(x) \cdot \frac{1}{x} + \ln x \cdot f'(x)$$

$$g'(4) = f(4) \cdot \frac{1}{4} + \ln 4 \cdot f'(4)$$

$$= \frac{6}{4} + 3 \ln 4$$

$$3) a) ROC_{avg} = \frac{A(90) - A(45)}{90 - 45} = \frac{405 - 190}{45} \text{ autos/min}$$

$$b) A'(95) = \lim_{h \rightarrow 0} \frac{A(95+h) - A(95)}{h}$$

$$\text{or} \\ \lim_{t \rightarrow 95} \frac{A(t) - A(95)}{t - 95}$$

$$A'(95) \approx \frac{A(100) - A(90)}{100 - 90} = \frac{205}{10} = 20.5$$

$$c) A(t) = 400 \text{ in } (70, 90)$$

$$\text{since } A(70) = 250 < 400$$

⋮

$$A(90) = 405 > 400$$

Then by IVT $A(t) = 400$.

$$d) f(t) = 227t^2 + 89t$$

$$f'(t) = 2(227)t + 89$$

$$\underline{f'(10) = 20(227) + 89}$$